Modified Gravity

1. Why and What

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Why Modify Gravity?

Because it’s there.

Theory Challenge – what freedom is there to create a consistent physical theory of gravity other than general relativity?

Experimental Challenge – we must test the extrapolation of physics from the solar system/compact object scales (10^{13} cm) by a factor of 10^{15} in length scale (and much more in scalar curvature) to cosmic scales.

Observational Challenge – what is responsible for cosmic acceleration?
1. Why and What?

Ways to modify gravity and ways not to. The many failures and why. What makes a consistent physical theory.

2. How?

Specific classes/models to modify gravity; specific ways to model independently modify gravity.

3. Where?

Where are we now? Where do we go next?
This course, at **Cosmology on the Beach**, focuses on cosmic gravity, not

**Lab tests**

- no model independent path to connect to cosmic gravity
- “dark energy in the lab” is a misnomer for dark sector physics, no clear connection to cosmos

**Dark matter replacement**

- Doesn’t compel me. Very tough to match all observations, e.g. offset from baryons, halo triaxiality, disk/bar stability.

**Screening mechanisms**

- See course by Prof. Mota
What is Modified Gravity?

No generally agreed definition!

Is it whatever causes cosmic acceleration? No, simple scalar field can do that.

Whatever causes cosmic acceleration through a mechanism other than its effective energy momentum tensor? [Not always easy to separate from Stückelberg transform]

Whatever modifies the tensor sector? [Not always easy to separate from Jordan/Einstein transform]

Whatever modifies the propagation of the spin 2 gravitron field? [Well defined, my favorite, ruled out!]
Is it Easy to Modify Gravity?

If modifying gravity is so vague, it must be easy to do, right?

And due to the discovery of cosmic acceleration, we must be the first generation to try, right?
Theoretical Landscape of the 20th Century: Competing Theories of Gravity – Slava Turyshev

Newton 1686  Poincaré 1890
Einstein 1912  Nordstrøm 1912  Nordstrøm 1913  Einstein & Fokker 1914  Einstein 1915
Whitehead 1922  Cartan 1923  Kaluza & Klein 1932  Fierz & Pauli 1939  Birkhoff 1943
Coleman 1983  Hehl 1997  Overlooked (20th century)

Theory must be:

- Complete: not a law, but a theory. Derive experimental results from first principles
- Self-consistent: get same results no matter which mathematics or models are used
- Relativistic: Non-gravitational laws are those of Special Relativity
- Newtonian: Reduces to Newton’s equation in the limit of low gravity and low velocities

Some authors proposed more than one theory, e.g. Einstein, Ni, Lee, Nordtvedt, Papapetrou, Yilmaz, etc.
Some theories are just variations of others
Some theories were proposed in the 1910s/20s; many theories in the 1960s/70s
Overlooked: this is not a complete list!
## A Long Road

**History repeats:** Theoretical soundness and Observational constraints rule out theories.

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- Will & Nordtvedt (1972) and Hellings & Nordtvedt (1972) are vector-tensor theories. Deviations can only be significant in high energy regime (e.g. Planck-scale energy).
- Yilmaz (1973) was mathematically inconsistent, but now is fixed. Does not predict black holes.
A Long Road

A certain role played by “aesthetics”

“Aesthetics-Based” Conclusion for 20th Century

Newton 1686  Poincaré 1890
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Slava Turyshev
Lovelock’s Theorem (1971)

The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant.

Go Arounds:

• Beyond second order
• Beyond locality
• Beyond the action principle
• Beyond 4D
• New degrees of freedom
**Modified Gravity**

Add new field content

- **Scalar**
  - Scalar-tensor & Brans-Dicke
  - Ghost condensates
  - Galileons
  - The Fab Four
  - KGB
  - Coupled Quintessence
  - Horndeski theories

- **Scalar-tensor** & Brans-Dicke

- **Vector**
  - Chern-Simons
  - Cuscuton
  - Chaplygin gases

- **Tensor**
  - Einstein-Aether
  - Lorentz violation
  - Massive gravity
  - Bigravity
  - TeVeS

- **Higher-order**
  - General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.

- **Higher-dimensional**
  - DGP
  - 2T gravity

- **Non-local**
  - Some degravitation scenarios
  - $f(R)$

- **Conformal gravity**
  - $f(G)$

- **Lorentz violation**

- **Emergent Approaches**
  - Padmanabhan thermo.
  - Padmanabhan thermo.

- **Strings & Branes**
  - Strings & Branes
  - Randall-Sundrum I & II
  - Kaluza-Klein
  - Generalisations of $S_{EH}$
  - Gauss-Bonnet
  - Lovelock gravity

- **Higher dimensions**
  - Einstein-Dilaton-Gauss-Bonnet
  - Cascading gravity

- **Add new field content**

- **Tessa Baker**

- **Some degravitation scenarios**

- **CDT**
Lovelock’s Theorem is powerful: GR cannot be other than it is (within assumptions).

Once we step beyond its light, the night is dark and full of terrors.

Worse, it may be arbitrary.

Is a new degree of freedom giving an arbitrary $f(R)$, dark energy coupling, $G(\phi, X)$, $f_{ij}$ any better than an arbitrary $V(\phi)$? Or better even than $\Lambda$?

I am not talking about a fine tuned mass scale (one number), but an arbitrary functional form.
Plus, technically unnatural.

One can partly protect against quantum corrections through, e.g. shift symmetries.

But the classes of gravity that used these as a central element are now (mostly) ruled out.

And we could use this for $V(\phi)$ too (e.g. PNGB).

What do we gain that is worth what we lose?

I proceed under the philosophy simply that we should test GR.
The Friedmann equations of motion arise from GR (with isotropy and homogeneity) and give superb agreement with early universe predictions, i.e. primordial nucleosynthesis and CMB.

For the recent universe, the premier method for testing cosmic gravity is comparing measures of cosmic expansion with measures of growth of large scale structure.

In GR, (linear) growth is completely determined by the expansion rate, i.e. Hubble parameter (and the split into matter density).
Gravity and Cosmic Structure

Modified gravity changes the strength of the gravitational coupling with time, and affects growth.

It can also change how matter moves, i.e. the relation between velocities and densities, affecting growth and e.g. redshift space distortions.

Furthermore, MG can change how light moves, i.e. the deflection law of gravitational lensing.

We’ll return to these in later lectures. Also see the courses of Profs. Font-Ribera, Mandelbaum, Padmanabhan.
Let us first examine what theoretical foundations we can stand on if we abandon GR.

1. Is it a theory?

Can we predict (in principle) what we need to? If one writes an ad hoc modification of the Friedmann equation, e.g.

\[ H^2 = \frac{8\pi G}{3} \rho_m + f(H, \dot{H}) \]

We have no idea what it means outside of expansion quantities, e.g. implications for growth.

This is not to say such phenomenology is useless, but one should be aware of its limits.
Sick Theories

2. Is it so sick we don’t want to touch it?

**Ghosts** – negative kinetic energy states that mean particles can run away from each other faster and faster (“like mad dogs across the universe”).

**Unbounded states** – Hamiltonian unbounded from below. The vacuum can spontaneously decay into negative energy particles.
3. Does the theory have uncomfortable elements?

Lorentz invariance violation

Superluminality

Instability

• **Tachyon/spinodal instability** – negative mass squared terms. May just involve a hilltop. Or a sign the field will fragment into clustered particles, e.g. oscillons.

• **Lagrange instability** – negative sound speed squared. Field perturbations grow rapidly, possibly ruining validity of calculation.
The model that revived scalar-tensor theories, \( f(R) \sim 1/R \) [Carroll, Duvvuri, Turner, Trodden 2004] was immediately (within 1 month) shown to be unstable.

The DGP [Dvali, Gabadadze, Porrati 2000] extradimensional theory was shown to have a ghost by 2005.
4. Does the theory have a well defined initial value formulation?

Theories involving higher derivatives require extra initial conditions.

If these are provided, there are extra Hamiltonian canonical variables, and generally there is the Ostrogradsky instability due to unbounded from below.

This can be avoided by extra constraints or degeneracies.

cf. beyond Horndeski and DHOST gravity
What is Modified Gravity (redux)?

It is not always easy to see whether gravity is really being modified.

**Jordan frame:**
\[
S = \int d^4x \sqrt{-g} \left[ R + f(R) + \mathcal{L}_m \right]
\]

- Minimal coupling to metric; geodesic deviation gives spacetime curvature

**Einstein frame:**
\[
S = \int d^4x \sqrt{-\tilde{g}} \left[ R(\tilde{g}) + \text{scalar} + \mathcal{L}_m(\tilde{g}) \right]
\]

- Nonminimal coupling to g; 5th force; spin 2 field separated (spin frame)
Scalar-Tensor Theory

f(R) relation to scalar-tensor gravity

\[ S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[ R + f(R) \right] + S_m[g_{\mu\nu}, \psi] \]

Rewrite with a Lagrange multiplier

\[ S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[ R + f(\lambda) + \frac{df}{d\lambda}(R - \lambda) \right] + S_m[g_{\mu\nu}, \psi] \]

EOM for \( \lambda \) gives \( \lambda = R \). Now conformal transform

\[ \tilde{g}_{\mu\nu} = (1 + f_R)g_{\mu\nu} \quad \phi = -M_P\sqrt{3/2} \ln(1 + f_R) \]

\( f_R \) defines a scalar field (called the scalaron) and the action takes the Einstein-Hilbert form

\[ S = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_P^2}{2} R - \frac{1}{2} \tilde{g}_{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_m[e^{\sqrt{2/3}\phi/M_P} \tilde{g}_{\mu\nu}, \psi] \]

with effective scalar potential

\[ V(\phi(R)) = \frac{M_P^2}{2} \frac{Rf_R - f}{(1 + f_R)^2} \]
Fully Modified Gravity

To fully, unambiguously modify gravity, one has to change the tensor propagation structure, i.e. the tracefree part of the tensor sector.

Effective field theories make this particularly clear.

This will change the speed of propagation of gravitational waves. This is now highly constrained by GW170817 and GRB170817A, and effectively ruled out (depending on how desperate you are to keep it).

“Fully modified gravity is ~dead.”

We return to this in Lecture 2.
Assorted Vocabulary

Conformal metric: \[ g_{\mu\nu} \rightarrow C(\phi, X) g_{\mu\nu} \quad ; \quad X \equiv (-1/2) g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi \]

Disformal metric: \[ g_{\mu\nu} \rightarrow g_{\mu\nu} + D(\phi, X) \partial_\mu \phi \partial_\nu \phi \]

Shift symmetry: \[ \phi \rightarrow \phi + c \quad ; \quad \text{obeyed by theories only depending on } \nabla \phi. \text{ Gives some protection against quantum corrections. Disformal theories are often shift symmetric. e.g. Galileons.} \]

Self-tuning: The scalar degree of freedom dynamically adjusts to cancel out a cosmological constant. Generally done through disformal terms. e.g. Fab 4, Fab 5.

The bad news: disformality is “ruled out” by \( c_T = 1 \).

We’re basically left with conformal theories (least interesting?).
Gravity Actions

**Einstein-Hilbert**

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \Lambda \right] \]

**Quintessence**

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \]

**K-essence**

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - K \left( \frac{1}{2} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) - V(\phi) \right] \]

**f(R)**

\[ S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[ R + f(R) \right] \]

**DGP**

\[ S = \frac{M_P^2}{2} \left[ \frac{1}{r_c} \int_{\text{bulk}} d^5x \sqrt{-g(5)} R^{(5)} + \int_{\text{brane}} d^4x \sqrt{-g} R \right] \]

**Galileons**

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2 R}{2} - \frac{1}{2} \sum_{i=1}^5 c_i \mathcal{L}_i - \mathcal{L}_m \right] \]

\[ \mathcal{L}_1 = M^3 \pi, \quad \mathcal{L}_2 = (\nabla_\mu \pi)(\nabla^\mu \pi), \quad \mathcal{L}_3 = (\Box \pi)(\nabla_\mu \pi)(\nabla^\mu \pi) / M^3 \]

\[ \mathcal{L}_4 = (\nabla_\mu \pi)(\nabla^\mu \pi) \left[ 2(\Box \pi)^2 - 2\pi;\mu\nu \pi;^{\mu\nu} - R(\nabla_\mu \pi)(\nabla^\mu \pi)/2 \right] / M^6 \]

\[ \mathcal{L}_5 = (\nabla_\mu \pi)(\nabla^\mu \pi) \left[ (\Box \pi)^3 - 3(\Box \pi)\pi;\mu\nu \pi;^{\mu\nu} + 2\pi;\mu;\nu;\pi;^{\mu;\nu} \pi;^{\rho} \pi;^{\rho;\mu} - 6\pi;\mu \pi;^{\mu\nu} \pi;^{\rho} G_{\nu\rho} \right] / M^9 \]

\[ S = \int d^4x \sqrt{-g} \left[ \left( 1 - 2c_0 \frac{\pi}{M_{\text{pl}}} \right) \frac{M_{\text{pl}}^2 R}{2} - \frac{c_2}{2} (\partial \pi)^2 - \frac{c_3}{M^3} (\partial \pi)^2 \Box \pi - \frac{c_4 \mathcal{L}_4}{2} - \frac{c_5 \mathcal{L}_5}{2} - \frac{M_{\text{pl}}}{M^3} c_G G^{\mu\nu} \partial_\mu \pi \partial_\nu \pi - \mathcal{L}_m \right] \]

and many more...
General Approaches to Modify Gravity

Next: how to approach modified gravity without choosing one particular theory.

• Effective field theory
• Property functions
• Modified Poisson equations

Within a particular theory, one can do computational simulations – see lectures by Prof. Mota.