Modified Gravity

2. How

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UC Berkeley

Essential Cosmology for the Next Generation 7
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This Course

1. Why and What?
Ways to modify gravity and ways not to. The many failures and why. What makes a consistent physical theory.

2. How?
Ways to model independently modify gravity.

3. Where?
Where are we now? Where do we go next?
General Approaches to Modify Gravity

How to approach modified gravity without choosing one particular theory:

• Effective field theory
• Property functions
• Modified Poisson equations
Effective Field Theory

Allow every possible operator in the action, consistent with chosen symmetries (e.g. Robertson-Walker background, 2nd order equations of motion).

Quadratic action (for first order perturbations) has 7 terms, each with time dependent coefficient.

Used in many fields, e.g. particle physics, dark matter, etc. [See Cliff Burgess, COTB 2012]. However cosmology works in a time dependent background, so coefficients are functions not numbers.

Only valid for linear theory, unless one goes to higher order, with more terms and coefficients.

Gubitosi, Piazza, Vernizzi 1210.0201
Bloomfield, Flanagan, Park, Watson 1211.7054
Gleyzes, Langlois, Piazza, Vernizzi 1304.4840
Bellini & Sawicki 1404.3713
Linder, Sengör, Watson 1512.06180
Effective Field Theory

\[ S_2 = \int d^4 x \sqrt{-g} \left[ \frac{m_0^2}{2} \Omega(t) R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2} \delta K \delta g^{00} \right. \]

\[ \left. - \frac{\bar{M}_2^2(t)}{2} \delta K^2 - \frac{\bar{M}_3^2(t)}{2} \delta K^\mu \delta K^\nu + \frac{\hat{M}^2(t)}{2} \delta R^{(3)} \delta g^{00} + m_2^2(t) \partial_i g^{00} \partial^i g^{00} + \mathcal{L}_m \right] \]

\[ R \rightarrow \text{Ricci scalar; note conformal factor } \Omega(t) \]

\[ K \rightarrow \text{extrinsic curvature} \]

\[ \Lambda \rightarrow \text{like scalar potential or cosmological constant} \]

\[ c \rightarrow \text{like scalar kinetic energy} \]

Fully modified gravity (changing the tensor propagator) comes from traceless part of dK, so \( \bar{M}_3^2(t) \)
Horndeski and Beyond

Horndeski gravity was not originally derived from EFT, but it is a special case with

\[ m_2 = 0; \quad 2\hat{M}^2 = \hat{M}_2^2 = -\hat{M}_3^2 \]

4 coefficients rather than 7.

Note that if the tensor propagator is not changed, then \( \hat{M}_3^2(t) = 0 \) and Horndeski is greatly reduced.

Horndeski has explicitly 2\textsuperscript{nd} order EOM, but EFT guarantees stability even if appears higher order.

This leads to “Beyond Horndeski” gravity and DHOST (degenerate higher order scalar-tensor) theory.

Horndeski does give clues to nonperturbative formulation.
Property Functions

Combinations of EFT terms enter into observational effects.

In particular some affect only the scalar sector, some only the tensor sector, and some link the two.

Property function approach [Bellini & Sawicki 2014] clarifies these physical properties by organizing terms in 4 time dependent functions (in Horndeski case).

\( \alpha_M \) – running Planck mass (coupling)
\( \alpha_K \) – kineticity: scalar kinetic structure
\( \alpha_B \) – braiding: mixing scalar and tensor sectors
\( \alpha_T \) – tensor wave speed deviation \((c_T^2-1)\)

All are functions of time, and 0 within GR.
EFT $\leftrightarrow$ Property Functions

**Translation table between EFT and property function**

For the gravitational slip, in the Newtonian limit the general expression is

$$\alpha_M = \frac{\dot{\Omega} + \frac{\dot{M}_2^2}{m_0^2}}{H\Omega + H\frac{M_2^2}{m_0^2}} = \frac{(\dot{p}/p)(1 + N) + \dot{N}}{H(1 + N)}$$

$$\alpha_K = \frac{2c + 4M_2^4}{m_0^2\left(H^2\Omega + H^2\frac{M_2^2}{m_0^2}\right)} = \frac{2}{H^2p(1 + N)}\left[c + 2M_2^4\right]$$

$$\alpha_B = -\frac{\bar{M}_1^3 + m_0^2\dot{\Omega}}{m_0^2\left(H\Omega + H\frac{M_2^2}{m_0^2}\right)} = \frac{-\dot{p} + \bar{M}_1^3}{Hp(1 + N)}$$

$$\alpha_T = -\frac{\bar{M}_2^2}{m_0^2\left(\Omega + \frac{M_2^2}{m_0^2}\right)} = -\frac{N}{1 + N}$$

\[N = \frac{-\alpha_T}{1 + \alpha_T}; \quad 1 + N = (1 + \alpha_T)^{-1} = c_T^{-2}\]

\[\frac{\dot{p}}{p} = H\alpha_M - \frac{\dot{N}}{1 + N}\]

\[\bar{M}_1^3 = -Hp\frac{\alpha_B}{1 + \alpha_T} - \dot{p} = -Hp\left(\frac{\alpha_B}{1 + \alpha_T} + \alpha_M - \frac{N'}{1 + N}\right)\]

\[c + 2M_2^4 = \frac{1}{2}\alpha_KH^2p(1 + N),\]

Linder, Sengör, Watson 1512.06180
Property Functions

Action in terms of property functions:

\[ \delta_2 S = \int d^3 x \ dt \ a^3 \frac{M(t)^2}{2} \left[ R^{(4D)} + \alpha_T(t) \delta_2 \left( \sqrt{h} R / a^3 \right) \right. \]

\[ + \alpha_K(t) H^2 \delta N^2 + 4\alpha_B(t) H \delta K \delta N \]

\[ \alpha_M \sim d \ln M^2 / H dt \]

Beyond Horndeski involves \( \alpha_H \), related to \( \hat{M}^2(t) \)

DHOST involves \( \alpha_L \), related to \( \tilde{M}_2^2(t) \)
## Mapping to Gravity Models

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<th>$\Omega$</th>
<th>$\Lambda$</th>
<th>$c$</th>
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### Model Class

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<th>Model Class</th>
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Horndeski Gravity

Translation between $\alpha_n$ and Horndeski $G_i(\Phi,X)$.

$$X \equiv -\frac{1}{2} g^{\mu\nu} \phi_\mu \phi_\nu$$

$$S = \int d^4x \sqrt{-g} \left[ \sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_m[g_{\mu\nu}] \right]$$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X) \Box \phi,$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) \left[ (\Box \phi)^2 - \phi_{;\mu\nu} \phi^{;\mu\nu} \right],$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \phi^{;\mu\nu} - \frac{1}{6} G_{5\mu}(\phi, X) \left[ (\Box \phi)^3 + 2 \phi_{;\mu} \phi_\nu^\alpha \phi_\alpha \phi_\mu - 3 \phi_{;\mu\nu} \phi^{;\mu\nu} \Box \phi \right]$$

$$M_\star^2 \equiv 2 \left( G_4 - 2 X G_{4X} + X G_{5\phi} - \dot{\phi} H X G_{5X} \right)$$

$$HM_\star^2 \alpha_M \equiv \frac{d}{dt} M_\star^2$$

$$HM_\star^2 \alpha_B \equiv 2 \dot{\phi} \left( X G_{3X} - G_{4\phi} - 2 X G_{4\phi X} \right) +$$

$$+ 8 X H \left( G_{4X} + 2 X G_{4XX} - G_{5\phi} - X G_{5\phi X} \right) +$$

$$+ 2 \dot{\phi} X H^2 (3 G_{5X} + 2 X G_{5XX})$$

$$M_\star^2 \alpha_T \equiv 2 X \left( 2 G_{4X} - 2 G_{5\phi} - (\ddot{\phi} - \dot{\phi} H) G_{5X} \right)$$

$$H^2 M_\star^2 \alpha_K \equiv 2 X \left( K_X + 2 X K_{XX} - 2 G_{3\phi} - 2 X G_{3\phi X} \right) +$$

$$+ 12 \dot{\phi} X H \left( G_{3X} + X G_{3XX} - 3 G_{4\phi X} - 2 X G_{4\phi XX} \right) +$$

$$+ 12 X H^2 \left( G_{4X} + 8 X G_{4XX} + 4 X^2 G_{4XXX} \right) -$$

$$- 12 X H^2 \left( G_{5\phi} + 5 X G_{5\phi X} + 2 X^2 G_{5\phi XX} \right) +$$

$$+ 4 \dot{\phi} X H^3 \left( 3 G_{5X} + 7 X G_{5XX} + 2 X^2 G_{5XXX} \right)$$
Galileon Gravity

\[ G_2 = c_1 \phi + \frac{c_2}{2} H^2 x^2 \]
\[ G_3 = -\frac{c_3}{2} H^2 x^2 \]
\[ G_4 = \frac{1}{2} + \frac{c_4}{4} H^4 x^4 \]
\[ G_5 = \frac{-3c_5}{4} H^4 x^4 + c_G \frac{\phi}{m_p} \]

\[ \alpha_T = c_T^2 - 1 = \frac{2\kappa_3}{\kappa_4} - 1 \]
\[ \alpha_B = \frac{2\kappa_5 x}{\kappa_4} \]
\[ \alpha_K = \frac{4\kappa_2 x^2}{\kappa_4} \]
\[ M_*^2 = \frac{-\kappa_4}{2} \]
\[ \alpha_M = \frac{d \ln M_*^2}{d \ln a} = \frac{\kappa_4'}{\kappa_4} \]

\[ x = (1/m_p) d\phi/d \ln a \text{ and } \bar{H} = H(a)/H_0 \]

\[ \alpha_B = \frac{2\kappa_5 x}{\kappa_4} \]
\[ = \frac{4c_3 \bar{H}^2 x^3 - 24c_4 \bar{H}^4 x^4 + 30c_5 \bar{H}^6 x^5 + 8c_G \bar{H}^2 x^2}{-2 + 3c_4 \bar{H}^4 x^4 - 6c_5 \bar{H}^6 x^5 - 2c_G \bar{H}^2 x^2} \]
\[ c_T^2 = \frac{2\kappa_3}{\kappa_4} \]
\[ = 1 + \frac{c_4}{2} H^4 x^4 + 3c_5 H^6 x^5 \left( \frac{H'}{H} + \frac{x'}{x} \right) - c_G H^2 x^2 \]
\[ 1 - \frac{3c_4}{2} H^4 x^4 + 3c_5 H^6 x^5 + c_G H^2 x^2 \]
Property Functions $\alpha(t)$

To proceed further, one must specify time dependence of $\alpha(t)$.

Regrettably common in the literature to see

$$\alpha_i^{\text{prop}}(a) = \bar{\alpha}_i \Omega_{\text{de}}(a)$$

but this is a very poor approximation!

It generically fails at $z < 10$ in all theory classes.

Beware! This appears as a default in HiCLASS code.

In EFT terms, this implies all action terms scale together, despite different powers of mass suppression and time dependences – it removes almost all EFT freedom.
Failure Illustrated

Simplest model: $f(R)$ only depends on one $\alpha$. It looks nothing like $\alpha_{\text{prop}}^i(a) = \bar{\alpha}_i \Omega_{\text{de}}(a)$

Planck mass running
1 parameter $f(R)$ gravity

$\alpha_m$

$\alpha_m / \Omega_{\text{de}}$

Today

Linder 1607.03113
“Tracker” Approximation

Another short cut used as an approximation all too often in the literature is called the tracker or attractor approximation.

\[ H \dot{\phi} = \text{constant} \equiv \xi H_0^2 \]

This is only generically valid in the de Sitter limit, i.e. in the future!

It throws away all the natural dynamics, i.e. the Klein-Gordon equation for \( \dot{\phi} \).

Proponents will claim that it is one possible evolution for \( \dot{\phi} \) and so a subclass of the full theory.

Such work is then neither model independent nor representative.
But it gets worse!

This approximation implies

\[ H^2 \rho_{\text{de}} = \text{constant} \]

Since \( \Omega_{\text{de}}(a) \propto \frac{\rho_{\text{de}}}{H^2} \)

the tracker condition forces

\[ \Omega_{\text{de}}(a) \propto \left( \frac{H_0}{H(a)} \right)^4 \]

in contrast to the cosmological constant fine tuning

\[ \Omega_{\Lambda}(a) \propto \frac{\rho_{\Lambda}}{H^2} \propto \left( \frac{H_0}{H(a)} \right)^2 \]

The fine tuning is at the \( 10^{-240} \) level vs \( \Lambda \)'s \( 10^{-120} \).

It’s not a tracker, it’s super fine tuned!
Functions of Time

Why is no simple time dependence expected?

Recall that the $\alpha$ are ratios of Lagrangian coefficients, e.g.,

$$c_T \equiv 1 + \alpha_T = \frac{1 + \frac{c_4}{2} H^4 x^4 + 3c_5 H^6 x^5 \left( \frac{H'}{H} + \frac{x'}{x} \right) - c_G H^2 x^2}{1 - \frac{3c_4}{2} H^4 x^4 + 3c_5 H^6 x^5 + c_G H^2 x^2}$$

Observables in turn are ratios of $\alpha$'s, e.g. the gravitational slip (difference between gravity strengths):

$$\bar{\eta} = \frac{(2 + 2\alpha_M)\left(\alpha_B (1 + \alpha_T) + 2(\alpha_M - \alpha_T)\right) + (2 + 2\alpha_T)\alpha_B'}{(2 + \alpha_M)\left(\alpha_B (1 + \alpha_T) + 2(\alpha_M - \alpha_T)\right) + (2 + \alpha_T)\alpha_B'}$$

Generally,

$$\text{Observable} \sim \frac{\sum \prod \left( \sum \frac{f_1(H,X)}{f_2(H,X)} \right)}{\sum \prod \left( \sum \frac{f_3(H,X)}{f_4(H,X)} \right)}$$

no simple time dependence!
Cosmic acceleration suggests that Einstein relativity may need modification.

How should we test cosmic gravity, other than one model at a time? How do we connect observations and theory in a model independent manner?

Note the expansion history $H(z)$ is merely one free function of time.

For cosmic structure we have 5 times as many! (kineticity, Planck mass running, braiding, tensor speed).

Now the tensor sector is as important as the scalar (matter) sector!
Unexpected synergies!

The tensor sector is accessible through gravitational waves: CMB B-modes, pulsar timing arrays, interferometers.

Galaxy surveys have deep complementarity with CMB surveys (and PTA, LIGO/LISA).
Gravitational Waves

Tensor constraints came sooner than expected!

GW170817 + GRB1070817A: synchronicity of GW and photon arrival within 2 seconds after signal propagation for 130 My (400 x 10^{13} s – thanks to Dark Energy Camera blind search) limits

\[ \frac{c_T}{c} - 1 < 10^{-15} \]

ruling out almost all theories of fully modified gravity!
**Implications of** $c_T = c$

Any theory with $c_T \neq c$ is essentially ruled out. In particular, this was the definition of “fully modified” gravity from Lecture 1!

Light follows null geodesics. $g_{\mu\nu} dx^\mu dx^\nu = 0$

If GW follows disformal $\rightarrow \Delta t$. $\mathcal{G}_{\mu\nu} dx^\mu dx^\nu = 0$

$$\mathcal{G}_{\mu\nu} = g_{\mu\nu} + D(\phi, X) \partial_\mu \phi \partial_\nu \phi$$

Only conformal theories survive.

$$\mathcal{G}_{\mu\nu} = C(\phi, X) g_{\mu\nu}$$
Implications of $c_T = c$

All disformal theories are gone, leaving only conformal theories (no more shift symmetry!).

\[ f(\phi)R + \text{scalar terms} \]

Horndeski with $G_5, G_4(X)$ (including Galileons) ruled out, much of Beyond Horndeski, all self tuning theories.

Personal view: conformal theories are unstable to quantum corrections and arbitrary in form. They are continuous with $\Lambda$CDM, not fully distinct. Some form of self tuning was our best hope to solve $\Lambda$ problem.

GR is looking more and more beautiful!
Some implications of $c_T = c$

$\alpha_T = 0$ doesn’t directly affect ghost condition but does simplify stability and observables (effective gravitational strength).

In particular, the condition for light and matter to feel the same gravitational strength becomes

$$\alpha_B = -2\alpha_M$$

New class: “No Slip gravity”

$$\frac{G_{\text{eff}}^\Phi}{G_N} = \frac{2m_p^2}{M^2} \frac{\alpha_B \left[ \alpha_B (1 + \alpha_T) + 2(\alpha_M - \alpha_T) \right] + \alpha'_B}{(2 - \alpha_B) \left[ \alpha_B (1 + \alpha_T) + 2(\alpha_M - \alpha_T) \right] + 2\alpha'_B} \rightarrow \frac{m_p^2}{M^2}$$

$$\bar{\eta} = \frac{(2 + 2\alpha_M) \left[ \alpha_B (1 + \alpha_T) + 2(\alpha_M - \alpha_T) \right] + (2 + 2\alpha_T)\alpha'_B}{(2 + \alpha_M) \left[ \alpha_B (1 + \alpha_T) + 2(\alpha_M - \alpha_T) \right] + (2 + \alpha_T)\alpha'_B} \rightarrow 1$$

Weaker gravity helps with fit to growth data.
The Richness of Gravity

In GR, expansion determines growth.

In modified gravity, cosmology is much richer. Plus the tensor sector!

We have learned to fit $H(z)$ with just a few parameters: $\Omega_m, w_0, w_a$.

Can we do the same with gravity functions?

Need close connection between theory, computation, and data to test/interpret the results.
Practical use of EFT/property functions, i.e. fitting time dependence, has not succeeded so far – and doesn’t look promising.

How should we connect theory to observations to learn about modified gravity?

Where do we go next? – Lecture 3

(and see lectures by Prof. Mota on simulations for a specific model)